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Development of Methods for Predicting Large Crack Growth in Elastic-Plastic Work-Hardening Materials in Fully Plastic Conditions

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Introduction and Objectives

This report is in fulfilment of Research Grant Reference NAGAW - 3909 for NASA Ames Research Center, Dryden Flight Research Facility, Edwards, CA 93523.

The objects of the first, exploratory, stage of the project were listed as:-

1. To make a detailed and critical review of the Boundary Element method as already published and with regard to elastic-plastic fracture mechanics, to assess its potential for handling present concepts in two-dimensional and three-dimensional cases. To this was subsequently added the Finite Volume method and certain aspects of the Finite Element method for comparative purposes.

2. To assess the further steps needed to apply the methods so far developed to the general field, covering a practical range of geometries, work hardening materials and composites: to consider their application under higher temperature conditions.

3. To re-assess the present stage of development of the energy dissipation rate, crack tip opening angle and J-integral models in relation to the possibilities of producing a unified technology with items 1 and 2.

4. To report on the feasibility and promise of this combined approach and, if appropriate, make recommendations for the second stage aimed at developing a generalised crack growth technology for its application to real-life problems.

Outline of the methods used

To keep this exploratory study within reasonable bounds, only monotonic loading of metals at room temperature has been used in the case studies. Seven such studies have been made, four of the selected problems computed by all four methods used, followed by three problems specific to one or two of the selected methods. The four methods were (1) The Boundary Element [B.E.]; (2) Finite Volume [F.V.]; (3) the Finite Element [F.E.] with a suite known as Modified WHAMSE [F.E(Wh)] and (4) the widely used ABAQUS program [F.E(Ab)]. A perhaps over-simplified statement on the differences in these approaches is that in the F.E. formulation the classical differential relationships for equilibrium and compatibility are discretised for each nodal point; in the F.V. formulation these relationships are discretised in their integrated form for each sub-region in the domain (or control volume); in the B.E. formulation they are discretised in an integrated form for the whole domain. The B.E. and F.V. Methods are reviewed in Appendices A and B of the full report.

All the programs are usable for linear elasticity and for incremental plasticity following the von-Mises criterion of yielding and the Prandtl-Reuss relationships, with work-hardening according to plastic potential theory, referred to hereafter as elastic-plastic.

The selected case studies

1. The cases-in-common to all methods used.

The purpose of the four cases-in-common is to give some measure of the effort (preparation and computing time) and accuracy or suitability of results around which to make a discussion of the methods. A simply supported beam is used, with a central load, spread as a uniform pressure over a small distance on each side of the central point of contact. Two cases are elastic - (a) un-notched and (b) notched (Figs. 1 and 2); and two elastic-plastic - (c) un-notched and (d) notched (Figs. 3 and 4). In all four cases plane strain is used. A coarse mesh "sighting shot" followed by medium and fine mesh computations were made to indicate convergence to about the 0.5% level.

Sample results for the four cases are given in Tables 1 to 4, using fine mesh computations only together with a "datum" derived from other sources. There is no reason to suppose that the "datum" values are any more reliable than the calculations made by the team, but at least they give an idea of the correspondence with the best calculated and experimental data available to us.

2. Further examples computed by particular methods are:

a) With the F.V. and F.E(Wh) methods, the same beam as in Fig. 4 was computed for an impact loading at a constant striker speed of 10m/s. The results are given in the full report.

b) Three-dimensional solutions of cases 1 to 4 were computed by F.E(WHAMSE), using a more refined mesh. The differences in values between the F.E(Wh) for the 3-D and 2-D cases are not large. The results are given in the full report.

c) Stable crack growth in an elastic-plastic beam. With the same beam as in Case 4, the stable growth of a crack was computed using F.E(Wh). Fig. 5 is a comparison of the computed and experimental load decay curves.

The work is discussed in detail in the full report.

Conclusions and recommendations for possible future work

For elastic problems of monotonic loading in fracture mechanics, there is little doubt that linear elastic fracture mechanics is universally accepted as an appropriate model. Although fatigue is predominantly elastic in terms of the applied stress, it is now generally accepted that, for short cracks in relation to the micro-structure, it is inadequate, and inhomogeneities of micro-structure dominate the formation and early growth of a fatigue crack.

For problems with plasticity there is less agreement but for many instances linear elastic fracture mechanics has been extended by the inclusion of a plastic zone correction factor.

This is not the place to discuss the present stage of research into fracture, but merely to point out that there is a strong case for further computational methods by which the considerations of the physical models can be more easily carried on.

With the further development of these computational methods, it is expected that attention can be given to objectives 3 and 4 during the extension of the work recommended below. It will therefore be important to keep a "watching brief" to incorporate such fracture studies at the right time. In particular, the J-integral procedure has been found wanting for both initiation and crack growth with extensive plasticity; initiation is being tackled by the Two Parameter methods, notably where crack tip fields are characterised by J and Q (hydrostatic stress); studies to describe crack growth by energy dissipation rate D, and associated crack tip opening angle (CTOA) exist, but are less well developed. We see a unification of J-Q and CTOA-D methods as the most promising approach and, as work proceeds, an additional project will be submitted to NASA when it is considered timely and promising to do so.

In this regard, there is a considerable amount of useful work that can be done using the Finite Element method and there are many research groups pursuing this route. It is contended that, on the evidence of this preliminary study, there are clearly benefits to be realised from developing other methods and the Boundary Element and Finite Volume techniques are particularly promising.

Our results show that the B.E. method is an excellent choice for linear elastic stress analyses, including fracture problems. While some of its advantages over other methods are lost when non-linear material behaviour is introduced, it shows considerable promise for further development for elastic-plastic continuum (but not for composites).

The Finite Volume method has potential for fracture problems, particularly non-linear such as in large plastic deformations or when they are dynamic. The method is also promising in regard to composite materials.

In view of our preliminary findings, it is recommended that:

1. **Boundary Elements**
   First, to explore the application of existing programs to two-dimensional elastic-plastic fracture mechanics problems. Attention would be given to the possible use of special crack-tip boundary elements to optimise the representation of the singular stress and strain fields there. There is also considerable scope for introducing iterative equation solution techniques of the congregate gradient type. The method should then be extended to three dimensional elastic-plastic problems.

   This would be a two-year project for two post doctorals working under Professor Fenner.

2. **Finite Volume method**
   Future work on the F.V. method should concentrate on modelling non-linear constitutive relationships and the development of large strain formulation in fracture and should look at the extension to composite materials.

   A one to two year project is envisaged.

3. **Fracture Mechanics**
   A continuing assessment of the progress of the computational work in order to start objectives 3 and 4 at the appropriate time.

Table 1

Computational Study No. 1 : Comparisons of fine mesh data only
Un-cracked beam; elastic; plane strain
Notation used (see Fig. 1)
q/P compliance of C relative to D

σ(max) maximum bending stress at A (for a unit load)

Results	B.E.	F.V.	F.E(Wh)	F.E.(Ab)	Datum
q/P mm/MN	31.09	30.72	31.17	31.15	31.03 [1]
σ(max) MPa	42.70	42.52	42.50	42.72	42.68*
DoF	272	832	1012	1002	
(DoF ratios	1	3.0	3.7	3.7)	
CPU time, s	0.3	59	100	10	
(CPU ratios	1	180	330	30)	
CPU(s)/DoF	0.0011	0.071	0.099	0.010	

[1] Underwood et al, 1985.
*using conventional engineering elastic beam theory with the central load spread over ±1mm as in the computations.

Table 2

Computational Study No. 2 : Comparisons of fine mesh data only
Cracked beam; elastic; plane strain
Notation used (see Fig. 2)
q/P the compliance of C relative to D
V/P the crack mouth opening compliance
2v/P the compliances of the crack profile at two selected positions
K_I the elastic crack tip intensity factor at unit load

Results	B.E.	F.V.	F.E.(Wh)	F.E.(Ab)	Datum
q/P mm/MN	5.26	5.28	5.18	5.24	5.10 [1]
V/P mm/MN	3.24	3.31	3.17	3.21	3.26 [2]
2/vP mm/MN					3.27 [3]
at a/32	0.385	0.433	0.37	0.381	0.394 [2]
at a/16	0.563	0.601	0.53	0.538	0.568 [2]
K_I MPa√m	0.936	0.988	0.964	0.942	0.953*
DoF	224	1322	6240	6018	
(DoF ratios	1	5.5	27.8	26.9)	
CPU time, s	1	182	900	64	
(CPU ratios	1	182	900	64)	
CPU(s)/DoF	0.0045	0.15	0.29	0.011	

[1] Underwood et al [1985]
[2] Gross et al [1968]
[3] Kapp et al [1985]
* using the lefm shape factor, Y = 10.650 from Srawley [1976], as listed by Towers [1981] in the form $K = PY/B\sqrt{W}$.

The wide differences in degrees of freedom (DoF) and Central processor units (CPU) times between the methods are to be noted. They differ roughly by one to two orders of magnitude between the methods. The two F.E. methods (F.E.(Wh) and F.E.(Ab)) have very similar degrees of freedom but differ in CPU by near 15 fold for reasons of the method of solution used in this version of WHAMSE. The F.E.(Wh) data have been generated here by using an existing three-dimensional mesh run in plane strain rather than by generating an actual two-dimensional mesh.

Table 3

Computational Study No. 3 : Comparison of fine mesh data only

Un-cracked beam; elastic-plastic; plane strain.

Notation used below; (see Fig. 4.3)

P(max) the final load at which a test run was stopped

q displacement of C relative to D

q_{pl} plastic displacement of C relative to D

q_{el} elastic displacement of C relative to D

σ(max) maximum bending stress at A.

Results	B.E.	F.V.	F.E(Wh)	F.E.(Ab)	Expt. [1]
P(max) kN	45.0	45.0	45.0	45.0	44.1*
σ(max) MPa	1296	1176	1247	1214	1310
q mm at P(max)	4.95	3.41	5.45	3.65	10.0
q_{pl}mm at P(max)	3.55	2.03	4.05	2.25	8.1
q_{el} mm at P(max)	1.40	1.38	1.40	1.40	1.9
q_{pl}mm at 44kN	3.47	0.84	2.71	0.97	8.0
q_{pl}mm at 40kN	0.95	0.44	0.55	0.48	2.7
DoF	344	1100	1908	2314	
(DoF ratios	1	3.2	5.5	6.7)	
CPU time, m	14	130	60	27	
(CPU ratios	1	9.3	4.2	1.9)	
CPU Case 3/Case 1	2800	130	35	160	

[1] Dagbasi [1989]. * A loading roller slipped at this point whilst the load was still increasing. A maximum estimated by extrapolation is about 45 to 46kN; the test was of course in between plane stress and plane strain.

Table 4

Computational Study No. 4 : Comparisons of fine mesh data only

Cracked beam; elastic-plastic; plane strain.

Notation used (see Fig. 4.4):-

q_{el}/P elastic compliance of C relative to D

V_{el}/P elastic crack mouth opening compliance

q, q_{pl} displacement of C relative to D and plastic component thereof

V, V_{pl} crack mouth opening displacement and plastic component thereof

Results	B.E.	F.V.	F.E(Wh)	F.E.(Ab)	Experiment [1] or Datum [2][3]
q_{el}/P mm/MN	6.24	6.25	6.15	6.20	6.60 [1] 6.63*; 6.03 [2]
V_{el}/P mm/MN	3.93	4.11	4.00	4.06	4.31 [1] 4.47*; 4.07 [3]
Max q mm	1.35	1.37	1.37	1.37	1.51 [1]
Max q_{pl} mm	0.02	0.02	0.04	0.03	0.03 [1]
Max V mm	0.868	0.905	0.896	0.904	0.955 [1]
Max V_{pl} mm	0.019	0.028	0.032	0.026	0.024 [1]
J (max) MN/m	0.163	0.133	0.143	0.136	0.159 [1]
DoF	264	1944	6190	4774	
(DoF ratios	1	7	23	18)	
CPU time, m	5	74	50	63	
(CPU ratios	1	15	8	13)	
CPU(s)/DoF	1.1	2.3	0.5	0.8	
DoF Case 4/Case 2	1.2	1.5	1.0	0.7	
CPU Case 4/Case 2	300	25	1.3	60	

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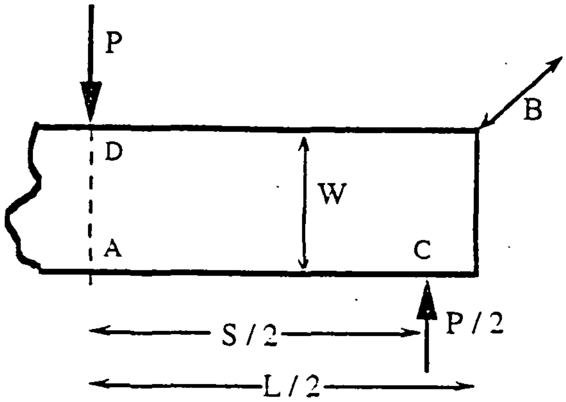
[1] Dagbasi, 1989. (Note the four computations here are made in plane strain whereas the beam tested is probably nearer plane stress: see above discussion and further results in Case 6. For the maximum load used in the computations, P = 216kN, a crack growth, $\Delta a \approx 0.5$mm is estimated from the experimental data whereas no growth was allowed in these studies).

[2] Underwood et al, [1985], *computed; plane stress and then plane strain.

[3] Wu Shang-Xian, [1984], *computed; plane stress and then plane strain.

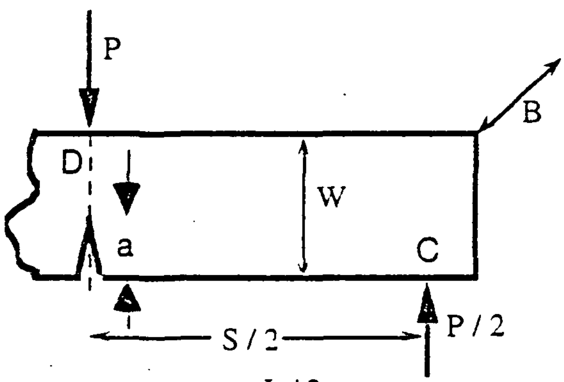


Dimensions :
S = 110 mm
W = 13.52 mm
B = 20.67 mm
L = 120 & 140 mm

Properties :
E = 2.0E+05 MPa
Poisson's ratio = 0.3

Load. P = 1 kN

Fig. 1

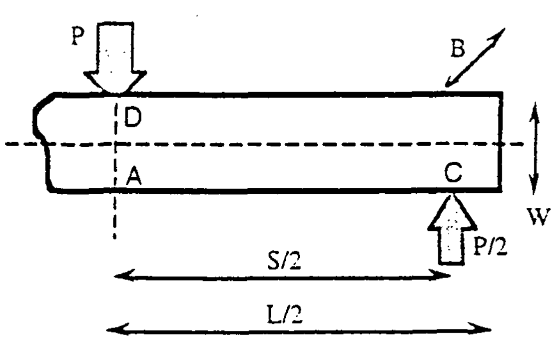


Dimensions :
S = 200 mm
W = 50 mm
B = 50 mm
L = 250 mm
a/W = 0.5

Properties :
E = 2.0E+05 MPa
Poisson's ratio = 0.3

Load, P = 1.0 kN

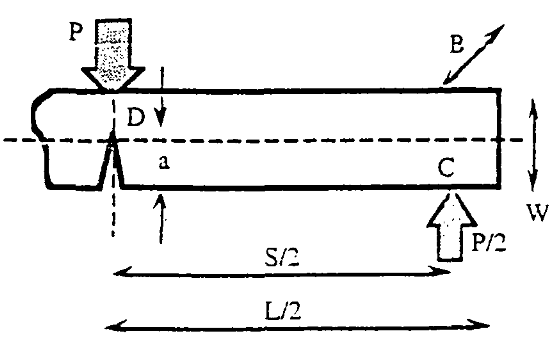
Fig. 2



Dimensions :

W = 13.52 mm
S = 110 mm
L = 120 mm
B = 20.67 mm

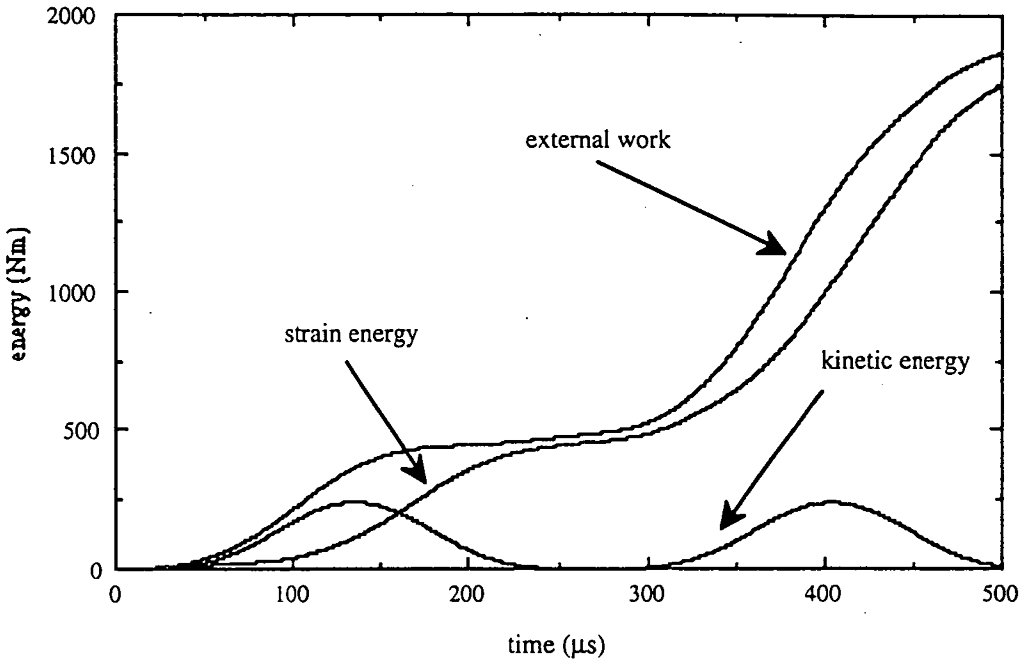
Fig. 3



Dimensions :

a = 50.77 mm
W = 94.2 mm
S = 4W = 376.8 mm
L = S+30 = 406.8 mm
B = 49.8 mm

Fig. 4



2000
1500
1000
500
0
energy (Nm)
0
100
200
300
400
500
time (μs)
external work
strain energy
kinetic energy

Fig. 5